

A two control volume model for the Thermal Lag Engine



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ABSTRACT

The Thermal Lag Engine (TLE) was proposed by Tailer in 1993 and its mechanical simplicity makes it an appealing technology to access renewable sources of energy or recover waste heat. This engine has a single moving part, the piston, which delivers power and acts as the prime mover of the working fluid. The TLE consists of a hot space with a high heat transfer capacity and a cold space with limited heat transfer capabilities. This engine is still not well understood. The paper presents a two control volume model for the TLE. In order to validate the model it is parameterized to describe a TLE built by Organ. The simulations of Organ's TLE experiments when compared to his experimental results have a correlation coefficient of 0.92 and are significantly correlated within the 95% confidence interval. The different characteristics of Organ's TLE are discussed based on the simulation results. Conclusions about the required characteristics of the heat transfer regimes and energy fluxes for the TLE are made. It is discussed that the key to a more efficient TLE will result from a careful design of the gas path and the cold heat exchanger, so that more of the enthalpy shuttled into the cold space can be converted into work.

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1. Introduction

Anthropogenic environmental degradation and fuel scarcity intensifies the need for economically viable technologies to access renewable sources of energy. Since 1807 air engines have provided an alternative technology for power generation due to their mechanical simplicity and fuel flexibility. James and Robert Stirling in 1816 included the regenerator into the air-cycle resulting in the patent of the Stirling engine. Other external combustion engines following Erickson cycles or Carnot cycles have also been built. Yet, it is Stirling technology that has demonstrated high thermodynamic and mechanical performance proving competitive for niche markets such as space applications. The Stirling refrigerator is widely used in cryogenic applications [15].

Stirling cycle prime movers and refrigerators are still an active area of research. Researchers are constantly proposing improvements to these machines and better ways to describe them, mostly concentrating on the classical configurations: alpha, beta and gamma. The three most challenging practical aspects of Stirling machines are: (1) lubrication and material selection for the moving parts in the hot section, (2) sealing the engine with a moving piston and (3) achieving heat exchangers with sufficient heat transfer capacity without increasing flow resistance or dead volume. The mechanical coupling between the piston and the displacer also

introduces mechanical complexities. To address these difficulties four different variants of Stirling-like machines have been developed since the 1960s, the thermoacoustic hybrid Stirling engine (TASHE) developed by Swift and his team [12], the free piston Stirling Engine developed by Beale [3], the pulse tube cycle initially proposed by Gifford and Longworth in 1964 [16] and finally the Thermal Lag Engine proposed by Tailer in 1993 [13]. These configurations reduce the moving parts, eliminating the mechanical coupling between the displacer and the piston, remove the moving parts in the hot section of the engine and address the sealing problem.

Tailer was granted a U.S. patent for the TLE in 1995 [14] and Fig. 1 illustrates one of his experimental engines. Tailer presents an elegantly simple idea for an external combustion engine, which uses a single moving part, the piston. The TLE is still not well described and understood, yet its mechanical simplicity makes it an appealing technology for solar applications or waste heat recovery. The purpose of this work is to show that it is possible to describe the TLE cycle through a two control volume model of the engine and to further the understanding of these machines by studying the simulations of the experiments presented by Organ in his book [11].

2. Organ and Tailer's TLE

In general the TLE consists of two spaces: a hot space where the heat to power the engine is supplied and a cold space into which

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Nomenclature

Suffixes refer to

h	hot control volume
c	cold control volume
p	piston
0	initial or fixed value
max	maximum value
f	flow path value
me	mean value
tot	total values of all the volumes
H and C	hot and cold sources characteristics
Differential notation	$\dot{f} = \frac{df}{dt}$

Variables and parameters

t	time
T	temperature (K)
P	pressure (Pa)
m	mass (kg)
V	volume (m ³)
A	area (m ²)

ρ	density (kg/m ³)
U	internal energy (J)
Q	heat transfer (J)
W	work (J)
H	enthalpy (J)
R	specific gas constant air (J/kg K)
c_v	constant volume heat capacity (J/kg K)
c_p	constant pressure heat capacity (J/kg K)
h	heat transfer coefficient (W/m ² K)
C_c	inner perimeter of piston chamber
C_f	flow contraction coefficient
A_p	Piston area (m ²)
D	diameter (m)
L	length (m)
d	thickness (m)
x	Piston displacement (m)
θ	crank angle starting at top dead center (Rad)
f	engine frequency (Hz)
$\dot{\theta}$	angular velocity (Rad/s)
r	Crank radius (m)
k_r	connecting rod length (m)

the piston brings the gas intermittently because of its motion. Thus, the piston takes the double function of delivering work and acting as the prime mover of the fluid in the engine. The TLE piston takes the role of the displacer in Stirling devices as its motion induces the movement of the gas through the different sections of the engine shifting in time within the cycle when the heat input and rejection occurs. Tailer and Organ have both built and measured TLEs, yet they have different claims about what are the mechanisms that generate the net heating and cooling of the gas at the right points in the cycle [13,11].

In Figs. 1 and 2 both Organ and Tailer's TLE configurations are illustrated. In both cases the engine consists of a hot space with a large surface area heat exchanger where heat is supplied, followed by an interface that separates the hot side from the cold side, allowing for the engine material to sustain a temperature difference. In the case of Organ this is a tube, in the case of Tailer this is a flame guard and water jacket. Then the cylinder top is a cooled surface, where the hot gas that reaches the piston can dump heat. Therefore the arrangement of components in these two engines is effectively the same. Yet there are some fundamental differences

between these machines. The cold heat transfer in Organ's machine is limited by the outside heat transfer coefficient, unlike Tailer's which uses water as the cooling medium. In Organ's TLE the cold heat exchanger is placed before the entry of the gas into the cylinder and the bore to stroke length (D_p/X_{max}) ratio is 1.3, Tailer does not have a cold heat exchanger there and (D_p/X_{max}) is 10. Thus different heat rejection behaviors can be expected for these engines. A further difference lies in the nature of the interface between heater and the cooler, which leads to the two different interpretations of how the TLE works. Tailer claims that the TLE should have a small as possible interface between the heater and cooler, and that the enthalpy shuttling between the two spaces is the important effect to consider [13]. Organ states that the TLE should have an optimal volume and hydraulic radius interface between the heater and the cooler. He refers to this interface as the pulse tube in accordance with his claim that the TLE is the inverse cycle of the pulse tube cryogenic coolers invented since the 1960s [11].

Whilst both Tailer and Organ provide a qualitative explanation for why their engines work, unfortunately Tailer's experiments are not sufficient to prove his claims and Organ's simulation does not correlate with his experimental results. Therefore the understanding of the thermodynamic phenomena that allows the TLE to produce work remains uncertain, and will only be unveiled through a thorough numerical and experimental investigation as Organ himself suggests [11]. For the Pulse Tube Engine (PTE) such investigations have been performed by Moldenhauer et al. [5,21].

3. Attempts to model the TLE

The previous attempts to model the TLE use a two or greater number of control volume description. The earlier attempts by Chen and West [8], West [18] and Wicks and Caminero [19] describe the engine as two volumes, one hot and the other cold. West presents the most insightful description of Tailer's engine and the TLE principle in general. In order to do so, he takes simplifying assumptions that allow him to identify states and processes within the cycle. Thus West presents a quantitative approximation of the TLE, indicates the importance of the time dependence of the processes in the engine and illustrates the unique effects that come from coupling an isothermal gas spring with other processes.

Wicks and Caminero simulate the TLE by fixing the heat transfer regimes inside the engine. They assume nominally constant heat

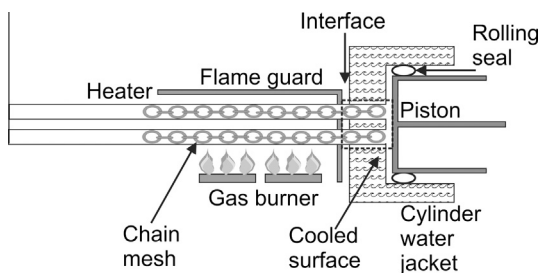


Fig. 1. Simplified schematic of Tailer's TLE.

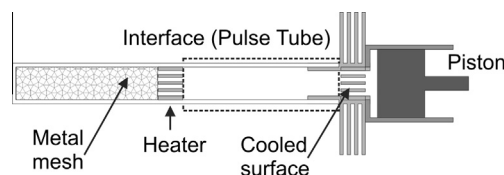


Fig. 2. Simplified schematic of Organ's TLE.

input in the hot section and sinusoidal heat output in the cold section. From this they assume that the variation of internal energy in the engine also follows a sinusoidal behavior and produce pV and TS diagrams. Yet the variation in internal energy is also coupled with the work output, and not taking this into account requires the use of unrealistic heat transfer regimes to describe the engine, as was observed in one of the simulation attempts made, and is also referred to in Wicks's paper [19].

Organ solves a one-dimensional approximation of the mass, momentum and energy balances in the engine. He does not attempt to model Taler's engine. Instead he develops a model to simulate the engine built and operated by his team, but presents no comparison between his simulations and his experimental results [11]. For his simulations Organ assumes laminar flow behaviour, and gives special attention to the interface region between the cold and hot space of the engine. He refers to this region as the pulse tube. He includes a comprehensive description of how the pulse tube permits work generation in a constantly heated and cooled engine. This is important in Organ's work because, unlike Taler he assumed the cylinder walls and piston head to be adiabatic, only transferring heat through the cold stack in the cylinder top. Interestingly there is not a pulse tube between the cold and hot space in Taler's engine. Nevertheless Organ claims that the TLE corresponds to the pulse tube engines and refrigerators family already patented since 1964 [11].

The concept of the Pulse Tube Engine (PTE) presented by Hamaguchi et al. [4] has been studied both numerically and experimentally. Yoshida et al. [20] in 2009 measured the work flux in a pulse tube engine showing they could be classified into the thermoacoustic standing wave engine group. In 2012 Moldenhauer et al. presented thorough numerical and experimental studies of the PTE reporting measured indicated powers and efficiencies of the order of 6W and 8% [6,5]. These results reaffirm the PTE's potential for low grade heat recovery applications.

The work presented here returns to West's simulation approach by simplifying the machine into two control volumes. As in West's and Wicks's work one is a hot volume with high heat transfer capacity and the other is a cold volume with limited heat transfer capacity and varying heat transfer area. Our modelling approach, unlike the previous attempts does not pre-establish the type of thermodynamic processes making up the cycle. On the other hand a dynamical modelling approach is taken; integrating energy, momentum and mass balances between the volumes in time. This model is the basis of the simulations later presented.

4. The model

In this model the engine volume consists of two spaces, one with fixed dimensions exposed to the hot source and one with varying dimensions exposed to the cold source. The total dimensions of the cold space's volume and heat exchange surface are determined by the piston's displacement. The displacement of the piston is as if it was coupled to a crankshaft and flywheel mechanism with constant angular velocity.

It is the general consensus that unlike thermoacoustic machines, the significant effects in the TLE are derived from bulk fluid motion [2]. Then it should be possible to describe the engine with the interaction of two control volumes with different heat transfer regimes coupled by an enthalpy flux. This is the general treatment given to the TLE models found in literature, with the exception of Organ who attempts to capture the effects arising from the temperature profiles in the gas and engine walls [11].

The temperatures of the hot source and cold sink are constant. The working fluid is air and is assumed to behave like an ideal gas. In most of the TLE engines measured and modelled that have been found in literature, the temperatures of operation of the gas are

such that the ideal gas behaviour is an adequate approximation. For air at the range of operation temperature and pressure of the engines there is less than 10% variation in c_p and c_v [9], thus allowing for these values to be treated as constants. The same approximations are taken by other authors.

The schematic of the model is presented in Fig. 3.

The TLE can be simplified in three main spaces, a heated space with a very large heat transfer capacity, a cooled space with varying volume and limited heat transfer capacity and an adiabatic space connecting the heater with the cooled space. The gas in the heater, because of its heat transfer capabilities, remains always close to the source temperature, whilst the gas leaving the heater into the adiabatic space takes a different thermal behaviour determined by the enthalpy flux from the heater, the change in volume of the engine and the heat rejection in the cooled space. Thus the volume of the adiabatic interface can be added into the cold volume.

In the model the volume in the cold CV is varied by the motion of the piston, which also varies the cold heat transfer area. The two volumes are connected by a flow area, through which enthalpy is shuttled between them. Mass, energy and momentum balances are applied to the control volumes performing a dynamical simulation. The internal energy of the gas in the volumes cycles in time, as the gas is brought out of equilibrium with its surroundings by the piston's motion. Fig. 3 illustrates the model's arrangement and from it the energy balances of the hot and cold volumes can be readily stated. The convention is that the energy fluxes leaving the volumes are taken to be negative. Then the energy balances for the CVs are:

For the hot volume:

$$dU/dt = \dot{Q}_h + \dot{H}_h \quad (1)$$

For the cold volume:

$$dU_c/dt = \dot{Q}_c - \dot{H}_h - \dot{W} \quad (2)$$

The dependence on the temperature gradient and the heat transfer coefficients are both of importance for the model to estimate the heat input and rejection. The exterior of the engine and the wall materials can be readily selected and modified to increase heat transfer capacity. Air cooled engines usually have their cooling capacity limited by the heat transfer coefficient of the exterior air. In the cases where the heat transfer outside is enhanced, the limitation to heat transfer will come from the internal resistances. Other authors have expressed the importance of estimating suitably the values of the heat transfer [11,19].

The heat exchange in the hot volume can be expressed as:

$$\dot{Q}_h = A_h h_h (T_H - T_h) \quad (3)$$

For the cold volume there is a fixed heat exchange area and a variable one, which is varied by the Piston's motion. The cold fixed area is usually composed of the cylinder head, the piston surface and in some cases a heat exchanger included after the interface

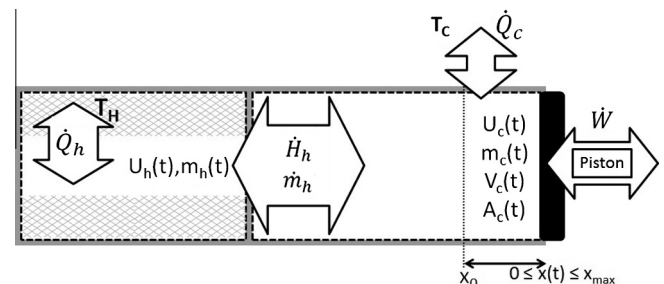


Fig. 3. Schematic of the two control volume model for the TLE.

region and before the piston chamber. Examples of inclusions that increase the fixed cold heat transfer area have been mentioned both by Tailer and Organ [14,11]. Even though within one control volume the heat transfer phenomena is averaged in space, special attention needs to be placed on what heat transfer mechanism is taking place in what surfaces, such that the magnitude of the heat transfer occurring can be approximated correctly. The piston, even if it is receiving an impinging velocity, is usually made of a poor conducting material and the heat transfer is limited by the outside heat transfer coefficient. Thus the piston area can be neglected or taken to have a lower heat transfer capacity. Further, in surfaces perpendicular to the predominant flow direction, because of flow separation generated by entry and exit effects the heat transfer is also limited. Therefore in the model some of these areas can be neglected or should have their heat transfer capacity limited with respect to other areas. These effects are introduced through a correction factor a_0 taking values between 0 and 1. The value of a_0 should be 0 when the heat transfer can be neglected, 1 to consider the complete heat transfer capacity and a middle value when the combined effect of the different heat transfer characteristics of these areas needs to be considered.

The inclusions before the piston tend to be flow through heat exchangers with a heat transfer area A_{xc} , and have a significant contribution to the heat rejection in the cycle. The usual position of this heat exchanger after the adiabatic space and before the cooled piston chamber makes it important to consider the temperature of the gas entering it. During the expansion this heat exchanger is receiving hot gas coming from the heater after flowing through the adiabatic space and in the compression it is receiving cold gas from the piston chamber. In order to describe this effect an extra term can be included into the cold heat transfer expression, where the temperature of the gas entering the heat exchanger corresponds to the hot temperature or when the adiabatic space exists to the mean temperature value for the whole engine T_{tot} during the expansion and to the cold volume temperature value T_c during the compression.

The piston motion exposes the chamber, that has a circumference C_c , such that the piston's displacement multiplied by the circumference gives the variable heat exchange area. In piston configurations where the bore D_p to stroke length X_{max} ratio is close to 1 or smaller, this variable lateral area contributes to the heat transfer dynamics that allows for work production in the engine, creating maximum heat rejection capacity at maximum engine volume. In the cases where D_p/X_{max} is much greater than 1 the flow in the piston takes a large radial component and other phenomena become the important effect, the heat transfer model presented here does not consider these cases. Then the heat transfer in the cold volume can be expressed by:

$$\dot{Q}_c = \begin{cases} (a_0 A_{0c} + C_c x) h_c (T_c - T_c) + A_{xc} h_c (T_c - T_{tot}) & \text{if } \dot{x} \geq 0 \\ (a_0 A_{0c} + C_c x) h_c (T_c - T_c) + A_{xc} h_c (T_c - T_c) & \text{if } \dot{x} < 0 \end{cases} \quad (4)$$

The value of T_{tot} is calculated by:

$$T_{tot} = \frac{U_h + U_c}{C_p m_{tot}} \quad (5)$$

The cold and hot internal heat transferred coefficients can be modelled using correlations found in literature taking time averaged mean values for the gas properties between the hot and cold CV. For the TLE the hot CV heat exchanger usually is a porous medium. There is no flow through the volume and thus the contribution of bulk flow velocities to the heat transfer phenomena can be neglected. Then the predominant heat transfer mechanism can be assumed to be natural convection in a medium of a small hydraulic radius. To estimate the coefficient value the Churchill and Chu correlation is used [17], where the characteristic length

for the correlation corresponds to the hydraulic diameter D_h of the porous medium of the hot heat exchanger. For the cold heat exchange coefficient the Annand correlation is used [1]. This correlation is specific to heat transfer mechanisms in cylinder-piston arrangements. In the TLE only gas compression and expansion processes take place, then the radiation term in the correlation can be neglected, and for initial approximations the correction factor a in the correlation can be taken to be the mean value of 0.5 [1].

Taking air standard analysis, the enthalpy fluxes can be expressed by:

$$\dot{H}_h = c_p \dot{m}_h T_{h/c} \quad (6)$$

Because mass is conserved between both volumes, the enthalpy flux term in the cold energy balance is just the opposite of the hot energy balance. The $T_{h/c}$ variable in Eq. (6) corresponds to the temperature of the CV from which the mass flux is coming. In the cases when the effect of the interface region needs to be considered and a better approximation is achieved when $T_{h/c}$ takes the value of T_{tot} instead of T_c during the compression. The direction of the mass flux is determined by the pressure difference between the volumes.

A mass flux model can be obtained by treating the flow between the volumes as if it was undergoing a sudden contraction or expansion between two spaces at different pressures. The mass flux model used to describe the mass transfer between the volumes in the TLE is derived from the integrated form of the momentum equation, which by making the following assumptions yields Eq. (7), where A_2 represents an area downstream of the flow from A_1 [17]:

1. The flow is considered to have a mean density for each time step.
2. The flow is steady between time steps.
3. The gas is always parting from an initial velocity of zero.
4. The flow is one dimensional.
5. The mass flux between the volumes will be modelled over the interconnecting flow area.
6. No dissipative losses are considered. They will be incorporated later using a loss coefficient C_f in the mass flux equation.
7. The flow has no change in gravitational potential energy.

$$A_2(P_1 - P_2) = \rho_{me}(A_2 v_2^2 - A_1 v_1^2) \quad (7)$$

From Eq. (7) a model can be derived to calculate the flow velocity for every time step proportional to the pressure differences between the hot and cold CVs. The value of the flow density corresponds to a linearisation of the density in space, performed by taking the mean density value between the volumes. Finally as pressure differences between the volumes range around 100 Pa, the flow is reciprocating and there is no flow through any of the volumes, it is possible to assume the initial velocity of every flow process to be zero. Changing to the appropriate nomenclature and taking the zero initial velocity assumption, Eq. (7) can be simplified to get the flow velocity between the hot and cold volumes. In the reciprocating case where the flow is changing direction as the pressure difference between the control volumes inverts, A_2 becomes A_f given that in one direction or the other, the flow is parting from the volume with the highest pressure, towards the flow area separating the two spaces. A_1 becomes A_0 because it would be the cross-sectional area of the hot or cold CV depending of the direction of the pressure gradient. As the initial flow velocity is assumed to be zero, A_0 disappears from the equation making the flow velocity only dependent on the flow area between the control volumes.

$$A_f(P_h - P_c) = \rho_{me}(A_f v_f^2 - A_0 v_0^2) \quad (8)$$

$$v_0^2 = 0$$

$$A_f \frac{(P_h - P_c)}{\rho_{me}} = A_f v_f^2$$

$$v_f = \sqrt{\frac{(P_h - P_c)}{\rho_{me}}}$$

To get to the mass flux equation this velocity needs to be multiplied by the flow area A_f for obtaining a volumetric flow rate and then by the mean density ρ_{me} . As the flow between the volumes can be occurring through some interface passage of smaller diameter than the piston bore, the gas is flowing into the piston chamber through a sudden expansion and out of it through a sudden contraction. Empirical correlations for sudden expansion and contractions introduce a correction factor C_f corresponding to the losses originated by the entry geometry. For the sudden expansion this value is taken to be 1 and for the sudden contraction 0.6 [7], by moving the value of this coefficient closer to zero further dissipative losses can be introduced into the model. Further, it has been shown experimentally that to use a mean density value between the two spaces for the flow density is an adequate approximation [7]. This then results in the expression for mass transfer used in the model, where the flux leaving the volume is considered negative.

$$\dot{m}_h = \begin{cases} -C_{fnc} \rho_{me} A_f \sqrt{\frac{(P_h - P_c)}{\rho_{me}}} & \text{if } P_h - P_c \geq 0 \\ C_{fch} \rho_{me} A_f \sqrt{\frac{(P_c - P_h)}{\rho_{me}}} & \text{if } P_h - P_c < 0 \end{cases} \quad (9)$$

This discrete equation is required because both the flow direction and the boundary conditions vary discretely with the direction of the pressure gradient. The values of P_c and P_h are also swapped between one instance of the equation and the other, thus the solution of the square root is always defined.

To complete the enthalpy transfer expression a discrete relationship is needed defining the temperature of the mass that is transferred and thus its enthalpy. This is expressed in Eq. (10).

$$dT_{h/c}/dt = \begin{cases} T_h & \text{if } P_h - P_c \geq 0 \\ T_c & \text{if } P_h - P_c < 0 \end{cases} \quad (10)$$

In the cases where the interface exists Eq. (10) becomes:

$$dT_{h/c}/dt = \begin{cases} T_h & \text{if } P_h - P_c \geq 0 \\ T_{tot} & \text{if } P_h - P_c < 0 \end{cases} \quad (11)$$

These conditional statements add numerical instabilities at the point when the pressure gradient shifts direction. This happens because at this point an abrupt change occurs in the first derivative of the mass flux, which is the second order derivative of the CV masses. On the other hand, they describe effectively the fact that hot mass comes into the cold volume from the heater and cold mass enters the heater from the interface or the piston chamber directly. For example in Tailer's engine, Eq. (10) is adequate, as the interface region between the hot and cold CV is not appreciable. Considering Organ's engine with a large interface between the heater and the cooler Eq. (11) is more appropriate.

Then, substituting Eqs. (3) and (6) in Eq. (1) as well as Eqs. (4) and (6) in Eq. (2) we have the following expressions for the internal energy of the hot and cold volume.

$$dU_h/dt = A_h h_h (T_H - T_h) + c_p \dot{m}_h T_{h/c} \quad (12)$$

$$dU_c/dt = \begin{cases} (a_0 A_{0c} + C_c \dot{x}) h_c (T_c - T_c) + A_{xc} h_c (T_c - T_{tot}) - c_p \dot{m}_h T_{h/c} - \dot{W} & \text{if } \dot{x} \geq 0 \\ (a_0 A_{0c} + C_c \dot{x}) h_c (T_c - T_c) + A_{xc} h_c (T_c - T_c) - c_p \dot{m}_h T_{h/c} - \dot{W} & \text{if } \dot{x} < 0 \end{cases} \quad (13)$$

From the definition of work, the shaft power term in Eq. (13) can be expressed in terms of the pressure in the cold chamber, the piston's area and the piston's velocity.

$$dU_c/dt = \begin{cases} (a_0 A_{0c} + C_c \dot{x}) h_c (T_c - T_c) + A_{xc} h_c (T_c - T_{tot}) - c_p \dot{m}_h T_{h/c} - P_c A_p \dot{x} & \text{if } \dot{x} \geq 0 \\ (a_0 A_{0c} + C_c \dot{x}) h_c (T_c - T_c) + A_{xc} h_c (T_c - T_c) - c_p \dot{m}_h T_{h/c} - P_c A_p \dot{x} & \text{if } \dot{x} < 0 \end{cases} \quad (14)$$

The model for the piston motion corresponds to the standard equations of a crankshaft and flywheel mechanism where the flywheel remains at a constant angular velocity:

$$x = k + r - \sqrt{k^2 - r^2 \sin^2 \theta} \quad (15)$$

Differentiating Eq. (15) results in:

$$\dot{x} = r \sin \theta \dot{\theta} + \frac{\dot{\theta} r^2 \sin \theta \cos \theta}{\sqrt{k^2 - r^2 \sin^2 \theta}} \quad (16)$$

The angular velocity $\dot{\theta}$ corresponds to the given engine frequency f in Hz:

$$\dot{\theta} = f 2\pi \quad (17)$$

The system consists of 10 variables. It can be solved by adding the perfect gas law, the definition of the internal energy and an expression for the conservation of mass, to Eqs. 9, 10, 12, 14, and 16. Through simple algebraic manipulations the pressure and the temperature can be expressed by the internal energy of the volumes. This procedure allows simplifying the integration to three variables: U_h, U_c, m_h . This guarantees mass conservation and reduces numerical error in the integration scheme. Eqs. (12) and (14) become:

$$dU_h/dt = A_h h_h (T_H - \frac{U_h}{c_v m_h}) + c_p \dot{m}_h T_{h/c} \quad (18)$$

$$dU_c/dt = \begin{cases} (a_0 A_{0c} + C_c \dot{x}) h_c (T_c - \frac{U_c}{c_v m_c}) + A_{xc} h_c (T_c - \frac{U_h + U_c}{c_v m_{tot}}) - c_p \dot{m}_h T_{h/c} - \frac{R U_c}{c_p (V_0 + x A_p)} A_p \dot{x} & \text{if } \dot{x} \geq 0 \\ (a_0 A_{0c} + C_c \dot{x}) h_c (T_c - \frac{U_c}{c_v m_c}) + A_{xc} h_c (T_c - \frac{U_c}{c_v m_c}) - c_p \dot{m}_h T_{h/c} - \frac{R U_c}{c_p (V_0 + x A_p)} A_p \dot{x} & \text{if } \dot{x} < 0 \end{cases} \quad (19)$$

Then Eqs. 9, 18 and 19 compose the differential system to be integrated. As the angular velocity of the flywheel is assumed constant, the velocity of the piston and the cold volume could be readily solved from Eq. (15)–(17).

5. The integration scheme

The system is integrated with a variable order solver specifically used for solving stiff differential equations. The integration variable is time, and the simulation code is designed to run until a steady state is reached, having a residual error between cycles below 1% of the indicated power calculated. The energy balances of the cycles are also checked to have a divergence smaller than this value. The code has been designed to be able to iterate through different parameter values and calculate the cycle indicators making it possible to study the influence of different parameters in the functioning of the engine.

6. Modeling Organ's experiments

The parametrization of Organ's TLE for the two control volume model is presented next. A comparison of the simulation results to Organ's experiment is presented in Fig. 5. The comparison demonstrates the validity of the modelling approach.

Fig. 4 shows the schematic of Organ's TLE and what regions of the engine are considered to be inside the hot and the cold control volumes. To simulate using two control volumes the pulse tube space was added to the cold control volume and the pulse tube

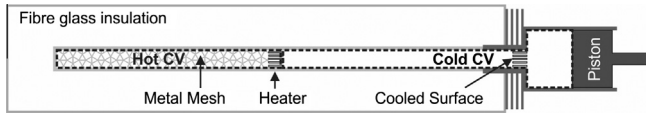


Fig. 4. Control volumes for Organ's TLE.

walls were considered to be adiabatic. The cold heat exchanger space was also added as a fixed volume to the cold CV. The varying cold heat transfer area in the piston chamber was also considered in the simulation in contrast with Organ's assumption of an adiabatic cylinder [11]. Due to the poor conductivity of the piston material the heat transfer through that surface was neglected. Considering the size of the interface between the heater and piston Eq. (11) was used to describe the enthalpy flux between the volumes. The parametrization used for Organ's engine in the simulation can be seen in Table 1.

In the case of Organ's TLE the largest resistance to the heat rejection is on the ambient side of the cold heat exchange surfaces. The external heat transfer coefficient was estimated to be 20 W/m² K, a standard value for a poor force convection or good free

convection coefficient for air. To introduce the effect of the external heat transfer coefficient the cold heat transfer model was changed to Eq. (20).

$$\dot{Q}_c = \begin{cases} \frac{1}{\frac{1}{h_{c,out}} + \frac{1}{h_c}} (a_0 A_{0c} + C_c \dot{x}) (T_c - T_c) + \frac{1}{\frac{1}{h_{c,out} A_{c,out}} + \frac{1}{h_c A_{c,c}}} (T_c - T_{tot}) & \text{if } \dot{x} \geq 0 \\ \frac{1}{\frac{1}{h_{c,out}} + \frac{1}{h_c}} (a_0 A_{0c} + C_c \dot{x}) (T_c - T_c) + \frac{1}{\frac{1}{h_{c,out} A_{c,out}} + \frac{1}{h_c A_{c,c}}} (T_c - T_c) & \text{if } \dot{x} < 0 \end{cases} \quad (20)$$

In his book Organ presents measurements of the shaft power of his TLE keeping a constant electrical power input and changing the engine's operation frequency. In order to describe the experiment a model for mechanical losses was introduced in the simulation using the standard analysis for slider crank mechanisms [10]. A comparison of the results of the model with Organ's experiments can be seen in Fig. 5. The experimental data has a correlation coefficient of 0.92 with the model, yielding a statistically significant correlation within the 95% confidence interval. This shows that the proposed modelling approach can be adequate to describe Organ's TLE experiments.

Table 1
Parameterization of Organ's TLE.

Parameter		Dimensions	Value
<i>Hot control volume</i>			
V_h	Hot volume	m ³	9.30×10^{-6}
P_e	Electrical input power	W	4.30×10^1
T_H	Hot source temperature	K	9.00×10^2
A_h	Hot heat exchange area	m ²	4.49×10^{-1}
h_h	Hot heat transfer coefficient at 9.5 Hz	W/m ² K	3.14×10^2
D_h	Hot heat exchanger hydraulic diameter	m	8.00×10^{-5}
k_i	Conductivity of the insulation material	W/m	4.00×10^{-2}
Th_i	Thickness of the insulation material	m	2.50×10^{-2}
<i>Cold control volume</i>			
X_0	Minimum displacement	m	1.00×10^{-3}
X_{max}	stroke length	m	2.47×10^{-2}
A_p	Piston head area	m ²	8.04×10^{-4}
V_{0c}	Minimum cold volume	m ³	1.14×10^{-5}
T_c	Cold sink temperature	K	3.00×10^2
A_{0c}	Piston chamber fixed heat transfer area	m ²	8.55×10^{-4}
A_{xc}	Cold heat exchanger area	m ²	1.62×10^{-3}
$A_{xc,out}$	Exterior cold heat exchanger area	m ²	1.55×10^{-2}
C_c	Cylinder circumference	m	1.02×10^{-1}
h_c	Cold heat transfer coefficient at 9.5 Hz	W/m ² K	7.02×10^1
$h_{c,out}$	Ambient heat transfer coefficient	W/m ² K	2.00×10^1
a	Annand correlation coefficient		5.00×10^{-1}
a_0	Fixed area correction factor		1.00×10^0
<i>Flow passage between the volumes</i>			
C_{hc}	Flow coefficient from hot to cold		1.00×10^0
C_{ch}	Flow coefficient from cold to hot		1.00×10^0
A_f	Flow area	m ²	5.03×10^{-5}
<i>Mechanical parameters</i>			
f	Engine frequency	Hz	9.50×10^0
k	Connecting rod length	m	6.00×10^{-2}
r	Crank length	m	1.24×10^{-2}
c_{fp}	Piston friction coefficient		2.00×10^{-1}
c_{fj}	Bearings friction coefficient		5.00×10^{-4}
<i>General parameters</i>			
T_{fill}	Filling temperature at BDC	K	3.00×10^2
P_{fill}	Filling pressure at BDC	Pa	1.00×10^5
c_p	Specific heat capacity at constant pressure	J/kg K	1.01×10^3
c_v	Specific heat capacity at constant volume	J/kg K	7.18×10^2
R	Gas constant	J/kg K	2.87×10^2

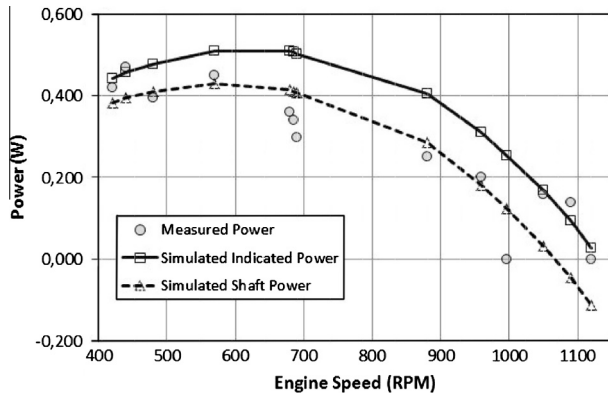


Fig. 5. Comparison of the measurements of Organ's Thermal Lag Engine and the two control volume model simulation [11].

7. Investigating Organ's TLE cycle

The model presented above can also be used to investigate the cycle behaviour of Organ's TLE. Cycle simulations have been performed using the previous model and parametrization. The engine was simulated at frequency values of 7 Hz, 9.5 Hz and 17.5 Hz corresponding to points in the experiments. The cycle indicators were calculated and the cyclic behaviour of the thermodynamic properties studied. From the description achieved with a two control volume model it is explained what factors can be considered for improving the performance of TLEs. Table 2 shows the cycle indicators for the different frequencies.

From Fig. 5 and Table 2 it can be seen that the engine performance is 5 times less with a 2-fold increase in frequency. The internal heat transfer resistance is reduced with increased piston velocity, yet the outside resistance remains unaltered limiting the heat rejection capability. Therefore, while the cycle period is reduced by a factor of 3 the heat transfer capacity is intensified only by 50%, resulting in an increase of the mean cycle temperature with engine frequency, as it can be seen in Fig. 7. Then the intensification in heat transfer observed at higher frequencies is mainly due to the increase in mean engine temperature. In this engine the limited heat rejection capacity is one of the causes for the poor performance. This can be appreciated in the indicated diagrams in Fig. 6. The hot CV has sufficient heat transfer capacity maintaining the same gradient for the expansion process at all frequencies. Yet the compression curves cross the expansion lines earlier in the cycle reducing work output as the engine speeds up. This effect is

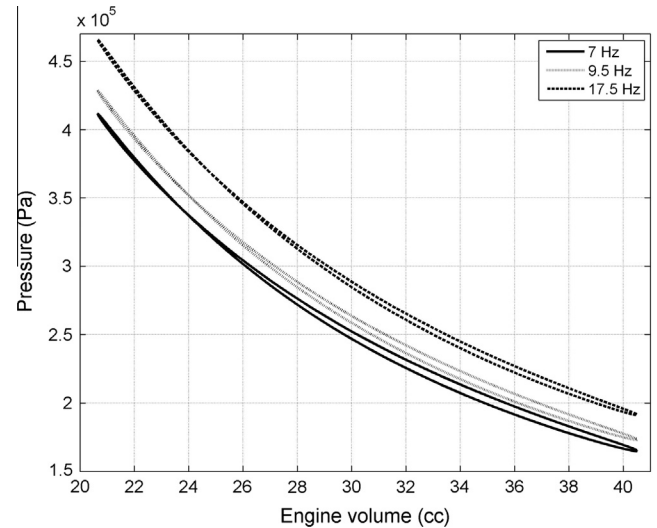


Fig. 6. Pressure vs. volume diagrams for Organ's TLE at different engine frequencies.

generated by the reduction in cycle period and the limited cold heat transfer capacity as West indicated in his theoretical work [18].

Therefore the external limit to heat rejection hinders engine performance. The effect of insufficient cooling can be best observed in Figs. 8 and 9, where the heat transfer and temperature dynamics in the hot control volume is presented. There it can be seen that heat delivery capacity in this volume allows the gas to maintain near isothermal behaviour as predicted by West and Chen [8,18]. Fig. 9 shows that when the piston reaches bottom dead center (BDC) the gas in the heater has reached the heater temperature. Then when heat rejection is insufficient, and as the heater delivers heat to the incoming cold gas during the compression, the temperature in the hot CV raises above the source temperature. For this part of the cycle the engine is pumping heat, attempting against power delivery. Using a regeneration strategy to recover this energy is inadequate as it does not make thermodynamic sense to convert work into heat for later recovery. Then as the heater with a large heat transfer capacity is a desired attribute in the engine, it seems a much better strategy to design the heat rejection mechanism such that this effect does not take place. Thus generating the triangular pV diagrams that West [18] predicted would maximize power delivery in a TLE.

Table 2
Cycle and engine indicators for Organ's TLE.

Indicators	Units	Values		
Engine frequency (f)	Hz	7	9.5	17.5
Heat transfer in hot CV (Q_h)	J	2.95	2.81	2.43
Heat transfer in cold CV (Q_c)	J	-2.89	-2.76	-2.42
ΔH from the hot volume	J	-2.95	-2.80	-2.43
Indicated work (W_i)	J	6.39×10^{-2}	5.37×10^{-2}	9.60×10^{-3}
Hot heat transfer coefficient (h_h)	W/m ² K	314	314	314
Cold heat transfer coefficient (h_c)	W/m ² K	56.7	70.3	108.0
Hot CV heat transfer capacity	W/K	141	141	141
Cold heat exchanger heat transfer capacity	W/K	7.09×10^{-2}	8.32×10^{-2}	1.12×10^{-1}
Piston chamber mean heat transfer capacity	W/K	3.21×10^{-2}	3.32×10^{-2}	3.64×10^{-2}
Indicated power (P_i)	W	4.48×10^{-1}	5.10×10^{-1}	1.68×10^{-1}
Shaft power (P_s)	W	3.88×10^{-1}	4.29×10^{-1}	3.29×10^{-2}
Indicated efficiency ($\eta = W_i/Q_h$)		2.17%	1.91%	0.40%
Volume per watt (V_{tot}/P_i)	cm ³ /W	91.5	79.4	241.0
Autot error	J	1.50×10^{-3}	1.90×10^{-3}	2.10×10^{-3}

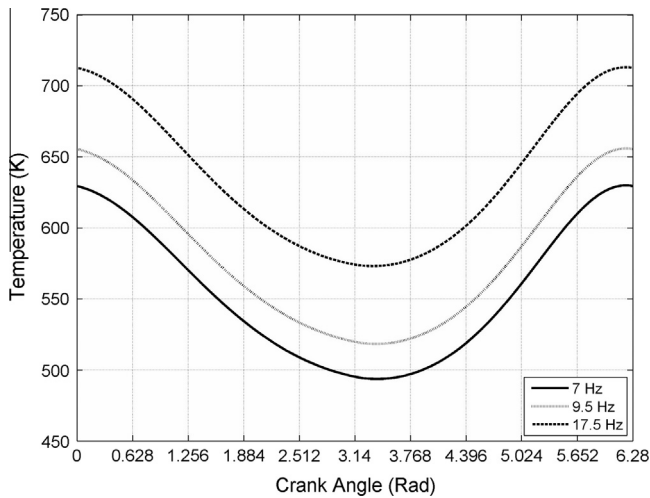


Fig. 7. Total temperature for Organ's TLE at different engine frequencies.

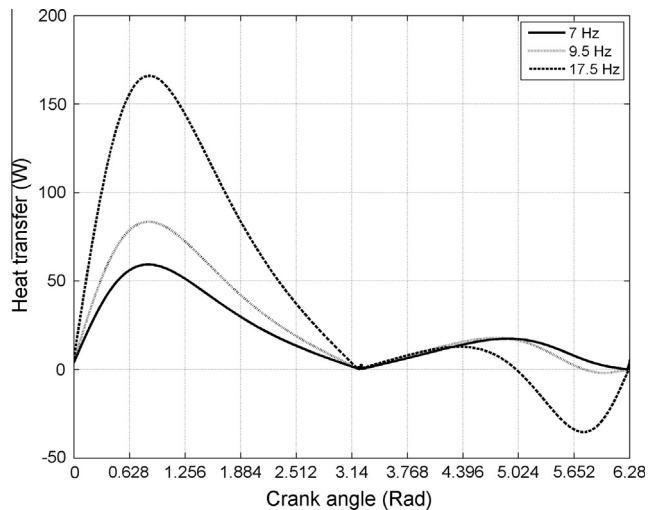


Fig. 8. Heat transfer in the hot control volume for Organ's TLE at different engine frequencies.

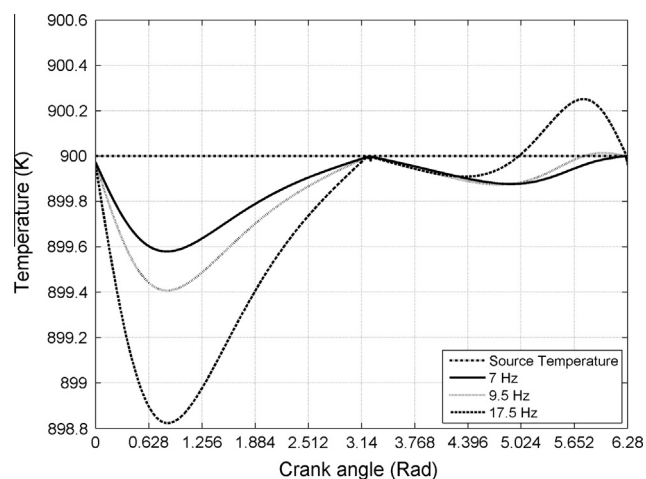


Fig. 9. Temperature in the hot control volume for Organ's TLE at different engine frequencies.

Simply dumping sufficient heat will improve engine performance. In general the limit to heat transfer in the engine is created by the external coefficient, yet enhancing external heat transfer can be readily done. Internally enhancing the heat transfer in the engine, especially in the piston chamber is difficult, postulating the biggest limitation for sufficient heat rejection. As in Organ, many TLE's include a cold heat exchanger before the piston where it is easier to enhance heat rejection. Yet in engines it is not only important how much heat is being dumped, but also when in the cycle the heat rejection occurs. When a heat exchanger is included before the piston, heat is rejected before it can be transformed into work hindering power delivery. For best performance maximum heat rejection should occur towards BDC. Even though Orgańs TLE is net heating towards top dead center (TDC) and net cooling towards bottom dead center (BDC), the cooling capacity in the gas path before the cylinder, governed by the cycle temperature dynamics, shifts maximum heat rejection towards TDC as seen in Fig. 10 and Fig. 11.

The cold heat transfer dynamics observed in Fig. 10 has the undesired effect of dumping the enthalpy delivered from the heater before it can be converted into work. This effect happens more intensely as the engine speeds up given that hotter gas is

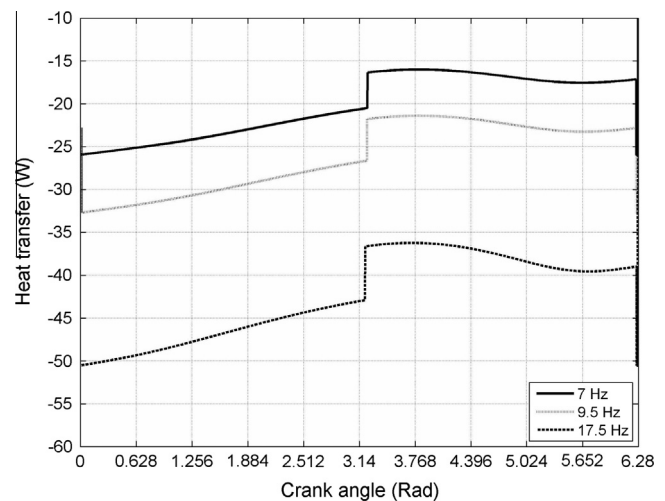


Fig. 10. Heat transfer in the cold control volume for Organ's TLE at different engine frequencies.

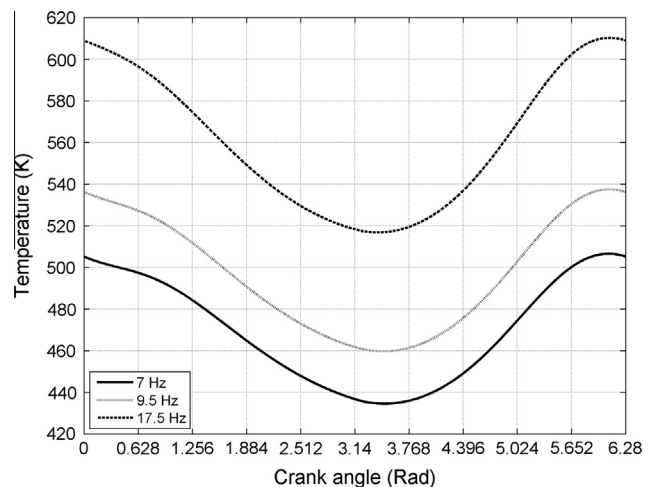


Fig. 11. Temperature in the cold control volume for Organ's TLE with different engine frequencies.

transported into the cold heat exchanger. This can be improved by eliminating the cold heat exchanger from the gas path before the heater. Then sufficient heat rejection can be attempted by enhancing heat transfer in the piston chamber. Considering that the variable exposure of the cold area in that space will shift maximum cooling towards BDC this is an ideal improvement. This is the original idea suggested by Tailer and developed by West [13,18].

Also the gas temperature dynamics in the hot CV is dominated by the heat transfer capacity of the heater, yet outside this space, the gas temperature dynamics is determined by the volume change. The cold heat exchangers always receives gas at lower temperatures towards BDC further reducing the heat transfer capabilities at that moment. If the cold heat exchanger was placed directly after the heater to change this, it would receive the hottest gas at the beginning of the expansion, thus simply incrementing the magnitude of the cooling at TDC and reducing even further the work generation. A way to improve that has been to separate the heater from the cold heat exchanger, for instance using the interface space. It is possible to optimize this space to achieve the right cyclic magnitude of heat rejection thus already improving engine performance, yet it might be difficult to use this strategy to shift in time when the maximum heat rejection occurs. Thus, in consequence with the removal of the cold heat exchanger from the gas entry into the piston chamber, it might be reasonable to eliminate the interface all together. This would allow reducing dead volume in the engine. Optimization of the heat transfer in the piston chamber could yield more satisfactory results.

Enhancing the heat transfer in the piston chamber to take advantage of the area variation is difficult. One idea could be to include micro-fins towards the BDC section of the chamber, guaranteeing sufficient heat transfer at right time in the cycle. Other authors have suggested different ways of achieving maximum heat rejection closer to BDC, including open ports [8] and optimizing the interface between the heater and cooler [11]. In the pulse tube engine variant other ideas to improve performance have been, to optimize the compression ratio, change the filling pressure and to include a bypass between the heater and the cold heat exchanger before the piston [5,4,20]. In the TLE a bypass could provide a parallel enthalpy path that does not pass through the cold heat exchanger delivering more energy directly for the expansion. Using a strategy like this may allow for optimizing the flow resistances such that only the desired amount of energy is dumped and more of the enthalpy shuttled between the volumes gets converted into work.

8. Conclusions

A two control volume model for the Thermal Lag Engine has been postulated. The work demonstrates that it is possible to handle Organ's pulse tube interface and simulate his experimental results with this approach. The simulations of Organ's TLE experiments, when compared to with his experimental results, have a correlation coefficient of 0.92 and are significantly correlated within the 95% confidence interval.

This model is used to simulate and study the cycle behaviour of Organ's TLE. It is concluded that while Organ's TLE has an adequate heating capacity, it has insufficient cooling capabilities limiting work production at higher frequencies. Also it is discussed that the heat rejection in this TLE happens at TDC and not when it is best at BDC.

The discussion shows that for the engine to perform well the hot heat exchanger should have the largest possible heat transfer capabilities, whilst the cold heat transfer should be limited to an optimal value and in time within the cycle.

Also it is concluded that the regeneration in the hot control volume, due to insufficient cooling, is an undesired effect and should be avoided.

The analysis gives that it would be beneficial, in order to increase power output and efficiency, to consider the gas path within the engine in order to shift the cooling capacity toward BDC. It is shown how the cold heat exchanger placed in the gas path before the piston does not attain this effect and hinders the performance of the engine, even if it allows guaranteeing the required magnitude of the heat rejection. Thus it is discussed that it might be beneficial to eliminate the cold heat exchanger before the piston completely and with it the pulse tube interface also reducing dead space. It is possible to shift the cold heat transfer to happen at BDC by having the piston's motion dominate the magnitude of the available heat transfer area. This allows concluding that the engine performance can be improved by enhancing and optimizing the heat transfer in the piston chamber.

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